MTH 201
Multivariable calculus and differential equations
Homework 2
Quadric surfaces and vector valued functions

1. Consider the planes
$$x + y + z = 1 \text{ and } x - 2y + 3z = 1.$$
   (a) Find parametric equation for the line of intersection of the planes.
   (b) Find the angle between the planes.

2. Consider the plane that passes through points P, Q, and R. Let S be the point **not** on this plane. Show that the distance d from S to the plane is (HW)
$$d = \frac{|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|}{|\mathbf{u} \times \mathbf{v}|},$$
where $\mathbf{u} = \overrightarrow{PQ}, \mathbf{v} = \overrightarrow{PR}$, and $\mathbf{w} = \overrightarrow{PS}$.

3. Sketch the surfaces
   (a) Elliptic paraboloid: $z = 4x^2 + y^2$
   (b) Hyperbolic paraboloid: $z = y^2 - x^2$.

4. Find the vector valued function that represents the curve of intersection of the circular cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$.

5. Find points of intersection of the helix $\overrightarrow{r(t)} = \langle \cos t, \sin t, t \rangle$ and the sphere $x^2+y^2+z^2 = 5$.

6. Find limits
   (a) $\lim\limits_{t\to 0} \left(e^{-10t}\mathbf{i} + \frac{t^2}{\sin^2 t}\mathbf{j} + \cos 2t\mathbf{k} \right)$
   (b) $\lim\limits_{t\to 1} \left(\frac{t^2-t}{t-1}\mathbf{i} + \sqrt{t}\mathbf{j} + \frac{\sin \pi t}{\log t}\mathbf{k} \right)$

7. Let $\overrightarrow{u(t)}, \overrightarrow{v(t)}$, and $\overrightarrow{w(t)}$ be differentiable vector valued functions.
   (a) Show that functions $t \to \overrightarrow{u(t)} \cdot \overrightarrow{v(t)}$ and $t \to \overrightarrow{u(t)} \times \overrightarrow{v(t)}$ are differentiable.
   (b) Find an expression for $\frac{d}{dt}\left[\overrightarrow{u(t)} \cdot \left(\overrightarrow{v(t)} \times \overrightarrow{w(t)}\right)\right]$.

8. Let $\overrightarrow{c(t)} = \langle |t|, |t - \frac{1}{2}| \rangle,\ t \in [-1, 1]$, describes the path of a billiard ball on a table. Find the distance travelled by the ball. (HW)

9. Find the arc length of the cycloid $\overrightarrow{c(t)} = \langle t - \sin t, 1 - \cos t \rangle,$ if $t \in [-1, 1]$.
   Note: The cycloid is traced out by a point moving on a rolling circle along a straight line.

10. Reparametrize the helix $\overrightarrow{r(t)} = \langle \cos t, \sin t, t \rangle$ with respect to arc length measured from $(1, 0, 0)$ in the direction of increasing t.

11. Suppose a particle moves along the curve given by $\overrightarrow{r(t)} = \langle 3\cos 2t, 3\sin 2t, 2t\rangle$. Find the position of the particle after travelling for a distance of $\frac{\pi\sqrt{10}}{3}$ units. (HW)

12. Show that
    (a) the curvature of Straight line is 0,
    (b) the curvature of a Circle of radius r is $1/r$. (HW)

13. Find unit tangent vector $\overrightarrow{T}$, principal unit normal vector $\overrightarrow{N}$, binormal vector $\overrightarrow{B}$, curvature κ, and torsian τ at a general point on the helix.

14. A particle moves with position function $\overrightarrow{r(t)} = \langle t, t^2, t^3\rangle$. Find the tangential and normal components of acceleration.